

BERTOTTI–ROBINSON GEOMETRY AND SUPERSYMMETRY**S. Ferrara^{*)}**Theoretical Physics Division, CERN
CH - 1211 Geneva 23**ABSTRACT**

The role of Bertotti–Robinson geometry in the attractor mechanism of extremal black holes is described for the case of $N = 2$ supersymmetry. Its implication for a model-independent derivation of the Bekenstein–Hawking entropy formula is discussed.

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BERTOTTI–ROBINSON GEOMETRY AND SUPERSYMMETRY

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ABSTRACT

The role of Bertotti–Robinson geometry in the attractor mechanism of extremal black holes is described for the case of $N = 2$ supersymmetry. Its implication for a model-independent derivation of the Bekenstein–Hawking entropy formula is discussed.

1. Extremal Black Holes and Attractors

In this report, I will discuss some recent work on the macroscopic determination of the Bekenstein–Hawking entropy-area formula for extremal black holes, using duality symmetries of effective string theories encoded in the supergravity low-energy actions. A new dynamical principle emphasizes the role played by “fixed scalars” and Bertotti–Robinson type geometries in this determination. Supersymmetry seems to be related to dynamical systems with fixed points describing the equilibrium and stability^a. The particular property of the long-range behavior of dynamical flows in dissipative systems is the following: in approaching the attractors the orbits lose practically all memory of their initial conditions, even though the dynamics is strictly deterministic.

The first known example of such attractor behavior in the supersymmetric system was discovered in the context of $N = 2$ extremal black holes^{1,2}. The corresponding motion describes the behavior of the moduli fields as they approach the core of the black hole. They evolve according to a damped geodesic equation (see eq. (20) in¹) until they run into the fixed point near the black hole horizon. The moduli at fixed points were shown to be given as ratios of charges in the pure magnetic case¹. It was further shown that this phenomenon extends to the generic case when both electric and magnetic charges are present². The inverse distance to the horizon plays the role of the evolution parameter in the corresponding attractor. By the time moduli reach the horizon they lose completely the information about the initial conditions, i.e. about their values far away from the black hole, which correspond to the values of various coupling constants, see Fig. 1. The recent result reported here³ is the derivation of the universal property of the stable fixed point of the supersymmetric

^aA point x_{fix} where the phase velocity $v(x_{\text{fix}})$ is vanishing is named a *fixed point* and represents the system in equilibrium, $v(x_{\text{fix}}) = 0$. The fixed point is said to be an *attractor* of some motion $x(t)$ if $\lim_{t \rightarrow \infty} x(t) = x_{\text{fix}}(t)$.

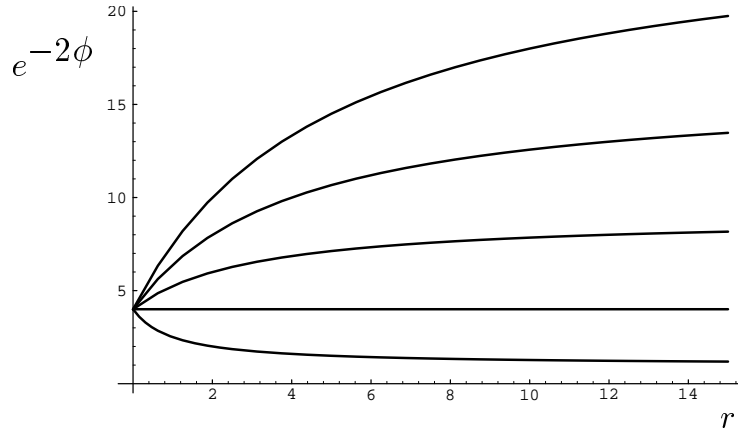


Figure 1: Evolution of the dilaton from various initial conditions at infinity to a common fixed point at $r = 0$.

attractors: fixed point is defined by the new *principle of a minimal central charge*^b and the area of the horizon is proportional to the square of the central charge, computed at the point where it is extremized in the moduli space. In $N = 2$, $d = 4$ theories, which is the main object of our discussion here, the extremization has to be performed in the moduli space of the special geometry and is illustrated in Fig. 1. This results in the following formula for the Bekenstein–Hawking entropy S , which is proportional to the quarter of the area of the horizon:

$$S = \frac{A}{4} = \pi |Z_{\text{fix}}|^2, \quad d = 4. \quad (1)$$

This result allows generalization for higher dimensions, for example, in five-dimensional space-time one has

$$S = \frac{A}{4} \sim |Z_{\text{fix}}|^{3/2}, \quad d = 5. \quad (2)$$

There exists a beautiful phenomenon in the black hole physics: according to the no-hair theorem, there is a limited number of parameters^c which describe space and physical fields far away from the black hole. In application to the recently studied black holes in string theory, these parameters include the mass, the electric and magnetic charges, and the asymptotic values of the scalar fields.

It appears that for supersymmetric black holes one can prove a new, stronger version of the no-hair theorem: black holes lose all their scalar hair near the horizon.

^bWe are assuming that the extremum is a minimum, as it can be explicitly verified in some models. However for the time being we cannot exclude situations with different extrema or even where the equation $D_i Z = 0$ has no solutions.

^cThis number can be quite large, e.g. for $N = 8$ supersymmetry one can have 56 charges and 70 moduli.

Black hole solutions near the horizon are characterized only by those discrete parameters which correspond to conserved charges associated with gauge symmetries, but not by the values of the scalar fields at infinity which may change continuously.

A simple example of this attractor mechanism is given by the dilatonic black holes of the heterotic string theory ^{4,5}. The modulus of the central charge in question which is equal to the ADM mass is given by the formula

$$M_{ADM} = |Z| = \frac{1}{2}(e^{-\phi_0}|p| + e^{\phi_0}|q|) . \quad (3)$$

In application to this case the general theory, developed in this paper gives the following recipe to get the area

i) Find the extremum of the modulus of the central charge as a function of a dilaton $e^{2\phi_0} = g^2$ at fixed charges

$$\frac{\partial}{\partial g}|Z|(g, p, q) = \frac{1}{2} \frac{\partial}{\partial g} \left(\frac{1}{g}|p| + g|q| \right) = -\frac{1}{g^2}|p| + |q| = 0. \quad (4)$$

ii) Get the fixed value of the moduli

$$g_{\text{fix}}^2 = \left| \frac{p}{q} \right|. \quad (5)$$

iii) Insert the fixed value into your central charge formula (3), get the fixed value of the central charge: the square of it is proportional to the area of the horizon and defines the Bekenstein–Hawking entropy

$$S = \frac{A}{4} = \pi |Z_{\text{fix}}|^2 = \pi |pq| . \quad (6)$$

This indeed coincides with the result obtained before by completely different methods^{5,6}.

In general supersymmetric $N = 2$ black holes have an ADM mass M depending on charges (p, q) as well as on moduli z through the holomorphic symplectic sections $(X^\Lambda(z), F_\Lambda(z))^{7-10}$. The moduli present the values of the scalar fields of the theory far away from the black hole. The general formula for the mass of the state with one half of unbroken supersymmetry of $N = 2$ supergravity interacting with vector multiplets as well as with hypermultiplets is⁷⁻¹⁰

$$M^2 = |Z|^2 , \quad (7)$$

where the central charge is⁷

$$Z(z, \bar{z}, q, p) = e^{\frac{K(z, \bar{z})}{2}} (X^\Lambda(z) q_\Lambda - F_\Lambda(z) p^\Lambda) = (L^\Lambda q_\Lambda - M_\Lambda p^\Lambda) , \quad (8)$$

so that

$$M_{ADM}^2 = |Z|^2 = M_{ADM}^2(z, \bar{z}, p, q) . \quad (9)$$

The area, however, is only charge dependent:

$$A = A(p, q) . \quad (10)$$

This happens since the values of the moduli near the horizon are driven to the fixed point defined by the ratios of the charges. This mechanism was explained before in ¹ and ² on the basis of the conformal gauge formulation of $N = 2$ theory⁸.

This attractor mechanism is by no means an exclusive property of only $N = 2$ theory in four dimensions. Our analysis suggests that it may be a quite universal phenomenon in any supersymmetric theory. It has in fact been extended to all $N > 2$ theories in four dimensions and to all theories in five dimensions¹¹. Further possible extensions to higher dimensions and to higher extended objects (p -branes) have also been discussed in recent literature¹².

In this report we will use the “coordinate free” formulation of the special geometry^{9,7,10} which will allow us to present a symplectic invariant description of the system. We will be able to show that the unbroken supersymmetry requires the fixed point of attraction to be defined by the solution of the duality symmetric equation

$$D_i Z = (\partial_i + \frac{1}{2} K_i) Z(z, \bar{z}, p, q) = 0 , \quad (11)$$

which implies,

$$\frac{\partial}{\partial z^i} |Z| = 0 \quad (12)$$

at

$$Z = Z_{\text{fix}} = Z \left(L^\Lambda(p, q), M_\Lambda(p, q), p, q \right) . \quad (13)$$

Equation $\partial_i |Z| = 0$ exhibits the *minimal area principle* in the sense that the area is defined by the extremum of the central charge in the moduli space of the special geometry, see Fig. 2 illustrating this point. Upon substitution of this extremal values of the moduli into the square of the central charge we get the Bekenstein–Hawking entropy,

$$S = \frac{A}{4} = \pi |Z_{\text{fix}}|^2 . \quad (14)$$

The area of the black hole horizon has also an interpretation as the mass of the Bertotti–Robinson universe ¹³ describing the near horizon geometry.

$$A/4\pi = M_{BR}^2 . \quad (15)$$

This mass, as different from the ADM mass, depends only on charges since the moduli near the horizon are in their fixed point equilibrium positions,

$$M_{BR}^2 = |Z_{\text{fix}}|^2 = M_{BR}^2(p, q) . \quad (16)$$

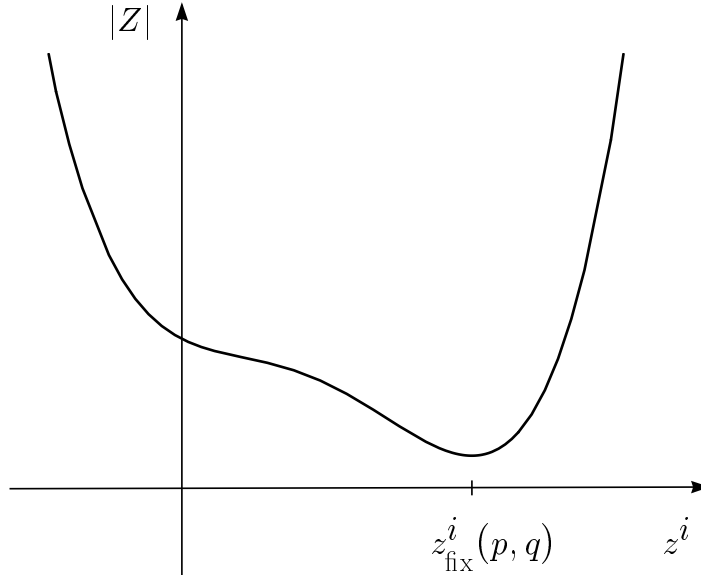


Figure 2: Extremum of the central charge in the moduli space.

Note that in the Einstein–Maxwell system without scalar fields the ADM mass of the extreme supersymmetric black hole simply coincides with the Bertotti–Robinson one, both being functions of charges:

$$M_{ADM}^2(p, q) = M_{BR}^2(p, q) . \quad (17)$$

We will describe below a near horizon black holes of $N = 2$ supergravity interacting with vector and hyper multiplets. The basic difference from the pure $N = 2$ supergravity solutions comes from the following: the metric near the horizon is of the Bertotti–Robinson type, as before. However, the requirement of unbroken supersymmetry and duality symmetry forces the moduli to become functions of the ratios of charges, i.e. take the fixed point values. We will describe these configurations, show that they provide the restoration of full unbroken $N = 2$ supersymmetry near the horizon. We will call them $N = 2$ attractors, see Sec. 2. We will briefly report on the extension of the attractor mechanism to more general (higher N and higher D) theories in Section 3.

2. Bertotti–Robinson Geometry and fixed scalars

The special role of the Bertotti–Robinson metric in the context of the solitons in supergravity was explained by Gibbons¹⁴. He suggested to consider the Bertotti–Robinson (\mathcal{BR}) metric as an alternative, maximally supersymmetric, vacuum state. The extreme Reissner–Nordström metric spatially interpolates between this vacuum and the trivial flat one, as one expects from a soliton.

Near the horizon all $N = 2$ extremal black holes with one half of unbroken supersymmetry restore the complete $N = 2$ unbroken supersymmetry. This phenomenon of the doubling of the supersymmetry near the horizon was discovered in the Einstein–Maxwell system in¹⁴. It was explained in¹⁵ that the manifestation of this doubling of unbroken supersymmetry is the appearance of a covariantly constant on shell superfield of $N = 2$ supergravity. In presence of a dilaton this mechanism was studied in¹⁶. In the context of exact four-dimensional black holes, string theory and conformal theory on the world-sheet the \mathcal{BR} space-time was studied in¹⁷. In more general setting the idea of vacuum interpolation in supergravity via super p-branes was developed in¹⁸.

We will show here using the most general supersymmetric system of $N = 2$ supergravity interacting with vector multiplets and hypermultiplets how this doubling of supersymmetry occurs and what is the role of attractors in this picture. The supersymmetry transformation for the gravitino, for the gaugino and for the hyperino are given in the manifestly symplectic covariant formalism^{9,10} ^din the absence of fermions and in absence of gauging as follows:

$$\begin{aligned}\delta\psi_{A\mu} &= \mathcal{D}_\mu\epsilon_A + \epsilon_{AB}T_{\mu\nu}^-\gamma^\nu\epsilon^B, \\ \delta\lambda^{iA} &= i\gamma^\mu\partial_\mu z^i\epsilon^A + \frac{i}{2}\mathcal{F}_{\mu\nu}^{i-}\gamma^{\mu\nu}\epsilon_B\epsilon^{AB}, \\ \delta\zeta_\alpha &= i\mathcal{U}_u^{B\beta}\partial_\mu q^u\gamma^\mu\epsilon^A\epsilon_{AB}C_{\alpha\beta},\end{aligned}\tag{18}$$

where λ^{iA} , $\psi_{A\mu}$ are the chiral gaugino and gravitino fields, ζ_α is a hyperino, ϵ_A , ϵ^A are the chiral and antichiral supersymmetry parameters respectively, ϵ^{AB} is the $SO(2)$ Ricci tensor. The moduli dependent duality invariant combinations of field strength $T_{\mu\nu}^-$, $\mathcal{F}_{\mu\nu}^{i-}$ are defined by eqs. (31), $\mathcal{U}_u^{B\beta}$ is the quaternionic vielbein¹⁹.

Our goal is to find solutions with unbroken $N = 2$ supersymmetry. The first one is a standard flat vacuum: the metric is flat, there are no vector fields, and all scalar fields in the vector multiplets as well as in the hypermultiplets take arbitrary constant values:

$$ds^2 = dx^\mu dx^\nu \eta_{\mu\nu}, \quad T_{\mu\nu}^- = \mathcal{F}_{\mu\nu}^{i-} = 0, \quad z^i = z_0^i, \quad q^u = q_0^u. \tag{19}$$

This solves the Killing conditions $\delta\psi_{A\mu} = \delta\lambda^{iA} = \delta\zeta_\alpha = 0$ with constant unconstrained values of the supersymmetry parameter ϵ_A . The unbroken supersymmetry manifests itself in the fact that each non-vanishing scalar field represents the first component of a covariantly constant $N = 2$ superfield for the vector and/or hyper multiplet, but the supergravity superfield vanishes.

The second solution with unbroken supersymmetry is much more sophisticated. First, let us solve the equations for the gaugino and hyperino by using only a part of the previous ansatz:

$$\mathcal{F}_{\mu\nu}^{i-} = 0, \quad \partial_\mu z^i = 0, \quad \partial_\mu q^u = 0. \tag{20}$$

^dThe notation is given in¹⁰.

The Killing equation for the gravitino is not gauge invariant. We may therefore consider the variation of the gravitino field strength the way it was done in^{15, 16}. Our ansatz for the metric will be to use the geometry with the vanishing scalar curvature and Weyl tensor and covariantly constant graviphoton field strength $T_{\mu\nu}^-$:

$$R = 0 \ , \quad C_{\mu\nu\lambda\delta} = 0 \ , \quad \mathcal{D}_\lambda(T_{\mu\nu}^-) = 0 \ . \quad (21)$$

It was explained in^{15, 16} that such configuration corresponds to a covariantly constant superfield of $N = 2$ supergravity $W_{\alpha\beta}(x, \theta)$, whose first component is given by a two-component graviphoton field strength $T_{\alpha\beta}$. The doubling of supersymmetries near the horizon happens by the following reason. The algebraic condition for the choice of broken versus unbroken supersymmetry is given in terms of the combination of the Weyl tensor plus or minus a covariant derivative of the graviphoton field strength, depending on the sign of the charge. However, near the horizon both the Weyl curvature and the vector part vanish. Therefore both supersymmetries are restored and we simply have a covariantly constant superfield $W_{\alpha\beta}(x, \theta)$. The new feature of the generic configurations which include vector and hyper multiplets is that in addition to a covariantly constant superfield of supergravity $W_{\alpha\beta}(x, \theta)$, we have covariantly constant superfields, whose first component is given by the scalars of the corresponding multiplets. However, now as different from the trivial flat vacuum, which admits any values of the scalars, we have to satisfy the consistency conditions for our solution, which requires that the Ricci tensor is defined by the product of graviphoton field strengths,

$$R_{\alpha\beta\alpha'\beta'}^{BR} = T_{\alpha\beta}\bar{T}_{\alpha'\beta'} \ , \quad (22)$$

and that the vector multiplet vector field strength vanishes

$$\mathcal{F}_{\mu\nu}^{i-} = 0 \ . \quad (23)$$

Before analysing these two consistency conditions in terms of symplectic structures of the theory, let us describe the black hole metric near the horizon.

The explicit form of the metric is taken as a limit near the horizon $r = |\vec{x}| \rightarrow 0$ of the black hole metric

$$ds^2 = -e^{2U} dt^2 + e^{-2U} d\vec{x}^2 \ , \quad (24)$$

where

$$\Delta e^{-U} = 0 \ . \quad (25)$$

We choose

$$e^{-2U} = \frac{A}{4\pi|\vec{x}|^2} = \frac{M_{BR}^2}{r^2} \ , \quad (26)$$

where the Bertotti–Robinson mass is defined by the black hole area of the horizon

$$M_{BR}^2 = \frac{A}{4\pi} \ . \quad (27)$$

We may show that this metric, which is the Bertotti–Robinson metric

$$ds_{BR}^2 = -\frac{|\vec{x}|^2}{M_{BR}^2} dt^2 + \frac{M_{BR}^2}{|\vec{x}|^2} d\vec{x}^2 , \quad (28)$$

is conformally flat in the properly chosen coordinate system. In spherically symmetric coordinate system

$$ds_{BR}^2 = -\frac{r^2}{M_{BR}^2} dt^2 + \frac{M_{BR}^2}{r^2} (dr^2 + r^2 d\Omega) . \quad (29)$$

After the change of variables $r = M_{BR}^2/\rho$ and $|\vec{x}| = M_{BR}^2/|\vec{y}|$ the metric becomes obviously conformally flat

$$ds_{BR}^2 = -\frac{M_{BR}^2}{\rho^2} dt^2 + \frac{M_{BR}^2}{\rho^2} (d\rho^2 + \rho^2 d\Omega) = \frac{M_{BR}^2}{|\vec{y}|^2} (-dt^2 + d\vec{y}^2) , \quad (30)$$

which is in agreement with the vanishing of the Weyl tensor.

Now we are ready to describe our solution in terms of symplectic structures, as defined in ¹⁰. The symplectic structure of the equations of motion comes by defining the $\text{Sp}(2n_V + 2)$ symplectic (antiselfdual) vector field strength $(\mathcal{F}^{-\Lambda}, \mathcal{G}_\Lambda^-)$.

Two symplectic invariant combinations of the symplectic field strength vectors are:

$$\begin{aligned} T^- &= M_\Lambda \mathcal{F}^{-\Lambda} - L^\Lambda \mathcal{G}_\Lambda^- \\ \mathcal{F}^{-i} &= G^{i\bar{j}} (D_{\bar{j}} \bar{M}_\Lambda \mathcal{F}^{-\Lambda} - D_{\bar{j}} \bar{L}^\Lambda \mathcal{G}_\Lambda^-) . \end{aligned} \quad (31)$$

The central charge as well as the covariant derivative of the central charge are defined as follows:

$$Z = -\frac{1}{2} \int_{S_2} T^- , \quad (32)$$

and

$$Z_i \equiv D_i Z = -\frac{1}{2} \int_{S_2} \mathcal{F}^{+\bar{j}} G_{i\bar{j}} . \quad (33)$$

The central charge, as well as its derivative, are functions of moduli and electric and magnetic charges. The objects defined by eqs. (31) have the physical meaning of being the (moduli-dependent) vector combinations which appear in the gravitino and gaugino supersymmetry transformations respectively. In the generic point of the moduli space there are two symplectic invariants homogeneous of degree 2 in electric and magnetic charges¹⁰:

$$\begin{aligned} I_1 &= |Z|^2 + |D_i Z|^2 , \\ I_2 &= |Z|^2 - |D_i Z|^2 . \end{aligned} \quad (34)$$

Note that

$$\begin{aligned} I_1 &= I_1(p, q, z, \bar{z}) = -\frac{1}{2} P^t \mathcal{M}(\mathcal{N}) P , \\ I_2 &= I_2(p, q, z, \bar{z}) = -\frac{1}{2} P^t \mathcal{M}(\mathcal{F}) P . \end{aligned} \quad (35)$$

Here $P = (p, q)$ and $\mathcal{M}(\mathcal{N})$ is the real symplectic $2n + 2 \times 2n + 2$ matrix

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \quad (36)$$

where

$$\begin{aligned} A &= \text{Im}\mathcal{N} + \text{Re}\mathcal{N}\text{Im}\mathcal{N}^{-1}\text{Re}\mathcal{N} , & B &= -\text{Re}\mathcal{N}\text{Im}\mathcal{N}^{-1} \\ C &= -\text{Im}\mathcal{N}^{-1}\text{Re}\mathcal{N} , & D &= \text{Im}\mathcal{N}^{-1} . \end{aligned} \quad (37)$$

The vector kinetic matrix \mathcal{N} was defined in Refs.⁷⁻¹⁰. The same type of matrix appears in (35) with $\mathcal{N} \rightarrow \mathcal{F} = F_{\Lambda\Sigma}$. Both \mathcal{N}, \mathcal{F} are Kähler invariant functions, which means that they depend only on ratios of sections, i.e. only on t^Λ, f_Λ ⁷⁻¹⁰.

The unbroken supersymmetry of the near horizon black hole requires the consistency condition (23), which is also a statement about the fixed point for the scalars $z^i(r)$ as functions of the distance from the horizon r

$$\frac{\partial}{\partial r} (z^i(r)) = 0 \quad \implies \quad D_i Z = 0 . \quad (38)$$

Thus the fixed point is defined due to supersymmetry by the vanishing of the covariant derivative of the central charge. At this point the critical values of moduli become functions of charges, and two symplectic invariants become equal to each other:

$$I_{1\text{ fix}} = I_{2\text{ fix}} = (|Z|^2)_{D_i Z=0} \equiv |Z_{\text{fix}}|^2 . \quad (39)$$

The way to explicitly compute the above is by solving in a gauge-invariant fashion eq. (38), yielding:

$$p^\Lambda = i(\bar{Z}L^\Lambda - Z\bar{L}^\Lambda) , \quad q_\Lambda = i(\bar{Z}M_\Lambda - Z\bar{M}_\Lambda) . \quad (40)$$

From the above equations it is evident that (p, q) determine the sections up to a (Kähler) gauge transformation (which can be fixed setting $L^0 = e^{K/2}$). Vice versa the fixed point t^Λ can only depend on ratios of charges since the equations are homogeneous in p, q .

The first invariant provides an elegant expression of $|Z_{\text{fix}}|^2$ which only involves the charges and the vector kinetic matrix at the fixed point $\mathcal{N}_{\text{fix}} = \mathcal{N}(t_{\text{fix}}^\Lambda, \bar{t}_{\text{fix}}^\Lambda, f_{\Lambda\text{ fix}}, \bar{f}_{\Lambda\text{ fix}})$.

$$(I_1)_{\text{fix}} = (|Z|^2 + |D_i Z|^2)_{\text{fix}} = -\frac{1}{2} P^t \mathcal{M}(\mathcal{N}_{\text{fix}}) P = (|Z_{\text{fix}}|^2) . \quad (41)$$

Indeed eq. (41) can be explicitly verified by using eq. (42). For magnetic solutions the area formula was derived in¹. This formula presents the area as the function of the zero component of the magnetic charge and of the Kähler potential at the fixed point^e.

$$A = \pi(p^0)^2 e^{-K} . \quad (42)$$

In the symplectic invariant formalism we may check that the area formula (42) which is valid for the magnetic solutions (or for generic solutions but in a specific gauge only) indeed can be brought to the symplectic invariant form¹:

$$A = \pi(p^0)^2 e^{-K} = 4\pi(|Z|^2 + |D_i Z|^2)_{\text{fix}} = 4\pi(|Z_{\text{fix}}|^2) = -2\pi p^\Lambda \text{Im} \mathcal{F}_{\Lambda\Sigma} p^\Sigma . \quad (43)$$

One can also check the first consistency condition of unbroken supersymmetry (22), which relates the Ricci tensor to the graviphoton. Using the definition of the central charge in the fixed point we are lead to the formula for the area of the horizon (which is defined via the mass of the Bertotti–Robinson geometry) in the following form

$$M_{BR}^2 = \frac{A}{4\pi} = (|Z|^2)_{D_i Z=0} , \quad S = \frac{A}{4} = \pi M_{BR}^2 . \quad (44)$$

The new area formula (44) has various advantages following from manifest symplectic symmetry. It also implies the principle of the minimal mass of the Bertotti–Robinson universe, which is given by the extremum in the moduli space of the special geometry.

$$\partial_i M_{BR} = 0 . \quad (45)$$

3. Attractors in more general theories

The previous analysis admits an extension to higher N theories at $D = 4$ and to $N \geq 2$ theories at $D = 5$. The general condition for getting a non-vanishing entropy-area formula is that extremal black-holes preserve only one supersymmetry (1/4 for $N = 4$ and 1/8 for $N = 8$). This condition is again obtained by extremizing the ADM mass in the moduli space. In this case the ADM mass is given, at $D = 4$, by the highest eigenvalue of the central charge matrix and supersymmetry implies that when the ADM mass is extremized, the other eigenvalues vanish. The vanishing of the other eigenvalues imply that the variation of the spin-1/2 partners of the gravitino vanishes on the residual unbroken (Killing) supersymmetry¹¹.

The entropy-area formula is always given by a U -duality invariant expression built out of the electric and magnetic charges^{20,21}. This is a consequence of the fact that both for $D = 4$ and 5 the area is given by an appropriate power of the extremized ADM

^eIn this paper we have a normalization of charges which is different from ¹ due to the use of the conventions of¹⁰ and not⁸.

mass which is a U -duality invariant expression (the central charge is an expression of first degree in terms of electric and magnetic charges)^{3,11}.

Similar arguments show that the attractor mechanism gives a vanishing (or constant) result for $D > 5$ since a U -duality invariant expression does not exist in that case. Similar considerations can be extended to higher p -extended objects in any D ¹².

Finally, we note that the mechanism of the doubling of supersymmetry at the attractor point is still operating in five dimensions²². The analogue of the Bertotti–Robinson geometry (which is $AdS_2 \times S^2$) is in this case the Tangherlini extremal $D = 5$ black hole²³ (with topology of $AdS_2 \times S^3$) which indeed admits two Killing spinors. This is the fixed moduli geometry of the $D = 5$ attractors.

Recently many applications of these ideas have been worked out, especially in the case of string theory compactified on three-dimensional Calabi–Yau complex manifolds²⁴. Determination of the topological entropy formula by counting microcosmic states in string theory, by means of D -brane techniques, has also been performed²⁵ and shown to give results, whenever obtainable, in agreement with the model-independent determination which uses the attractor mechanism.

Finally it should be mentioned that several properties of “fixed scalars” have been investigated²⁶. In particular, it has been shown that the attractor mechanism is also relevant in the discussion of black hole thermodynamics out of extremality²⁷.

4. Acknowledgements

It was a pleasure to give this talk on the occasion of the 65th birthday of Prof. Bruno Bertotti. The material covered in this talk comes mainly from work done jointly with Renata Kallosh and Andrew Strominger, both of which I would like to thank for very pleasant and fruitful collaboration.

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